Branes, Orientifolds and the Creation of Elementary Strings

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Abstract

The potential of a configuration of two Dirichlet branes for which the number of ND-directions is eight is determined. Depending on whether one of the branes is an anti-brane or a brane, the potential vanishes or is twice as large as the dilaton-gravitational potential. This is shown to be related to the fact that a fundamental string is created when two such branes cross. Special emphasis is given to the D0-D8 system, for which an interpretation of these results in terms of the massive IIA supergravity is presented. It is also shown that the branes cannot move non-adiabatically in the transverse direction. The configuration of a zero brane and an orientifold 8-plane is analyzed in a similar way, and the implications for the type IA-heterotic duality and the heterotic matrix theory are discussed.

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1   Introduction

It has become apparent in the last two years that Dirichlet-branes (D-branes) [1, 2] are the "solitonic" objects of string theory which are relevant for the non-perturbative regime of the theory [3, 4]. These D-branes can be studied in a variety of ways, and this has proved very useful. For example, D-branes can be analyzed using techniques of perturbative string theory, either in terms of suitable boundary conditions of open strings, or as coherent states in the closed string sector. It is also possible to study the field theory on the world-volume of the D-brane, and D-branes can be analyzed from the point of view of the low-energy supergravity theory. Many calculations have been performed using different techniques, and this has allowed for a number of consistency checks.

In this paper we shall analyze a certain class of configurations of two D-branes which is characterized by the property that the number of mixed boundary conditions of an open string stretching between them is ND= 8. It is well known that a system of two D-branes retains unbroken supersymmetries if ND= $0, 4$ or 8 [5]. The cases ND= 0 and 4 have been analyzed from a number of different points of view [2, 6, 7], and it has been found that two such branes do not exert a force onto each other. Furthermore, there exist solutions to the supergravity equations which reproduce all of these configurations. On the other hand, the case ND= 8 is much less understood, and there exist contradictory claims in the literature. For example

- The string calculation predicts that the force vanishes for ND= 8 configurations [8, 6, 9]. In the open string description there is a contribution from the sector $R(-1)^F$, which is absent for all other D-brane configurations. From the closed string point of view, the cancellation occurs between the NS-NS and the R-R sector. The contribution in the R-R sector is somewhat puzzling as one does not expect, for example, a D0-brane and an D8-brane to interact through a R-R gauge field.

- There are solutions of the supergravity equations of motion which preserve a quarter of the supersymmetries and look like the brane configurations of ND= 8 [10]. For the case of the D0-D8 system, however, there exists no solution.

- A Yang Mills analysis of the D0-D8 system in [11] shows that there is a non-zero force between D0-branes and D8-branes, and a non-zero force between D0-branes and an orientifold-8-plane (Ω8-plane). The forces cancel if eight D8-branes (and their images) are at the orientifold plane. The system in [11] was used successfully to explain certain aspects of the $E_8 \times E_8$ heterotic matrix model [12, 13, 14]. These results correspond to the string calculation if only the NS-NS sector contribution is taken into account.

- From the point of view of the supergravity, the force on the Dp-brane in the background of a Dp'-brane, where $p' > p$, can be calculated. For ND= 8, the force which is due to the exchange of the graviton and the dilaton is repulsive.

- A careful treatment of the D0-D8 system using massive supergravity theory shows that this system is inconsistent unless extra macroscopic elementary strings are introduced that end on the D0-brane [15].

We will show that indeed in the case of ND= 8 the string calculation is correct. The Yang-Mills analysis correctly accounts for the short distance degrees of freedom, but misses a linear potential coming from the R-R sector. For the case of the D0-D8 system, this potential can be interpreted as coming from an elementary string, which is thus in agreement with [15]. The calculation also indicates that an elementary string is created when the two branes cross each other adiabatically, and this is related to the Hanany-Witten effect [16] by a series of dualities.

We analyze the velocity dependent force in the D0-D8 system, and we find, somewhat surprisingly, that the branes cannot move in the transverse direction non-adiabatically. This has important implications for the non-perturbative behavior of type IIA string theory in the presence of D8-branes: the theory appears to be ten-dimensional at strong coupling. This is consistent with the fact that there does not exist a massive eleven-dimensional supergravity [17].

The paper is organized as follows. In section 2, we explain the open string theory calculation, and show that a fundamental string is created when the two branes cross; we also relate this to the Hanany-Witten effect. In section 3, we explain how the supergravity analysis of Polchinski and Strominger [15] gives an interpretation for the R-R contribution force in the D0-D8 case. In section 4, we analyze the configuration of the D0-brane and the Ω8-plane, and in section 5, we determine the velocity dependent potential for the D0-D8 and the D0-Ω8 configurations. In section 6 we show that our results are consistent with the world-line theory point of view, and we explain some of the implications for the matrix models. We have included an appendix, where the results are derived from a closed string point of view.

While this paper was being finalized, the papers [30, 31, 32] appeared in which some overlapping results have been obtained.

2 Stationary Potential in the ND= 8 system

Let us consider a stationary configuration of two parallel or orthogonal D-branes, and suppose that we have chosen the coordinates of our spacetime so that the branes are parallel to the coordinate axes. Consider an open string that is stretched between the

two branes. This string satisfies for every coordinate a Neumann (N) or Dirichlet (D) boundary condition at either end. We denote by NN the number of coordinates for which both ends have a N condition, by DD the number of coordinates for which both ends satisfy a D condition, and by ND the number of coordinates for which one end has a D, and one end a N condition. As we are considering the ten-dimensional superstring, NN+ND+DD= 10. We shall always assume that the time direction x^0 is a NN direction, so that we are considering D-branes rather than D-instantons. The D-branes will be separated along the DD directions by a vector $\mathbf{R}$, and the free energy of such a configuration is given at one loop by the annulus amplitude

$$\mathcal{A} = 2V_{NN} \int \frac{d^{NN}k}{(2\pi)^{NN}} \int_0^\infty \frac{dt}{2t} \mathrm{Tr}\left[e^{-2\pi\alpha' t(k^2+M^2)}(-1)^{F_S}\frac{1}{2}(1+(-1)^F)\right], \qquad (2.1)$$

where V_{NN} is the spacetime volume of the NN-directions, F_S and F are the spacetime and worldsheet fermion numbers, respectively, and

$$M^2 = \frac{R^2}{4\pi^2\alpha'^2} + \frac{1}{\alpha'}\left\{\sum_n(\alpha_{-n}\cdot\alpha_n + nb_{-n}c_n + nc_{-n}b_n) + \sum_m m(\psi_{-m}\cdot\psi_m + \beta_{-m}\gamma_m - \gamma_{-m}\beta_m) + a\right\}.$$
$$(2.2)$$

Here $a = 0$ in the Ramond (R) sector, and $a = -1/2 + \mathrm{ND}/8$ in the Neveu-Schwarz (NS) sector. The moding of the bosonic and fermionic oscillators $(\alpha_n^\mu, \psi_m^\mu)$ in the NN and DD directions is given by

$$n \in \mathbb{Z} \quad , \quad m \in \left\{ \begin{array}{ll} \mathbb{Z} & \mathrm{R} \\ \mathbb{Z}+1/2 & \mathrm{NS} \end{array} \right. , \qquad (2.3)$$

and in the ND directions by

$$n \in \mathbb{Z}+1/2 \quad , \quad m \in \left\{ \begin{array}{ll} \mathbb{Z}+1/2 & \mathrm{R} \\ \mathbb{Z} & \mathrm{NS} \end{array} \right. . \qquad (2.4)$$

The moding of the ghost and superghost oscillators is always as in (2.3), so that their contribution will cancel against the bosonic and fermionic contributions in two NN or DD directions.

Let us concentrate on the case D$p \perp$D$(8-p)$ (ND= 8). This includes the D-particle D8-brane system, the configuration of a D7-brane and an orthogonal D-string, $etc.$ Let us denote by x^9 the transverse DD direction, and let R be the transverse distance between the two branes along x^9. The potential between the branes is then

$$V_{Dp\perp D(8-p)}(R) = -2\int \frac{dk_0}{2\pi} \int_0^\infty \frac{dt}{2t} \mathrm{Tr}\left[e^{-2\pi\alpha' t(k_0^2+M^2)}(-1)^{F_S}\frac{1}{2}(1+(-1)^F)\right]. \qquad (2.5)$$

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Integrating over k_0 and performing the traces gives [6]

$$
\begin{aligned}
V_{Dp \perp D(8-p)}(R) &= -\int \frac{dt}{2t} (8\pi^2 \alpha' t)^{-1/2} e^{-R^2 t/(2\pi\alpha')} \left[\frac{f_2^8(q) - f_3^8(q) \pm f_4^8(q)}{f_1^8(q)} \right] \\
&= -\frac{1}{2} T_0 R \left[1 \mp 1 \right],
\end{aligned} \tag{2.6}
$$

where $q = e^{-\pi t}$, and the functions f_i are defined in appendix A. Here $T_0 \equiv (2\pi\alpha')^{-1}$ is the string tension, and we have used the "abstruse identity". The three terms in the bracket correspond to the traces over the NS, R and $R(-1)^F$ sector, respectively, and the sign of the third term corresponds to the choice of the action of $(-1)^F$ on the R-sector ground states. The trace over $NS(-1)^F$ vanishes identically, as the fermionic oscillators in the transverse directions $\mu = 1, \dots, 8$ have zero modes. The R-sector, on the other hand, has only fermionic zero modes in the $\mu = 0, 9$ directions; their contribution cancels against the (bosonic) superghost zero modes, and the $R(-1)^F$ trace does contribute.[1] The contribution of this sector is independent of q, implying that only the massless string modes, rather than the full string spectrum, contribute.

The open string one-loop calculation can be related, by a modular transformation, to a closed string tree-level calculation (see, for example, appendix A). Under this transformation, the NS and R contributions come from the (closed) NS-NS sector, and the $R(-1)^F$ contribution from the (closed) R-R sector. The former represents (in the large R limit) the combined interaction of the graviton and the dilaton which is repulsive in this case.[2] The sign of the latter contribution depends on whether we are considering two branes or two anti-branes, or one brane and one anti-brane.

It follows from (2.6) that the vacuum energy of a brane-brane system differs from that of the corresponding brane-anti-brane system. In one case the vacuum energy vanishes, whereas in the other it is equal to $-T_0 R$. Let us now consider the effect of rotating the $D(8-p)$-brane by π in the plane spanned by a direction of the $D(8-p)$-brane and the transverse direction x^9; this turns the $D(8-p)$-brane into an anti-brane ($\overline{D(8-p)}$-brane). On the other hand, this operation is topologically equivalent to moving the Dp-brane across to the other side of the the $D(8-p)$-brane, and we conclude that the sign in (2.6) flips as the Dp-brane moves from one side to the other.

Suppose that the system is in the vacuum state of vanishing energy when the Dp-brane is to the left of the $D(8-p)$-brane. As we move the Dp-brane adiabatically to the right across the $D(8-p)$-brane, the system remains in the same state. As explained above however, the vacuum on the right hand side is different. The difference in energy is manifested in the creation of a string between the branes (see figure 1).

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[1]This is actually true for any system with NN+DD= 2, in particular ND= 8, but also for boundary conditions involving background worldvolume gauge fields for $\mu = 1, \dots, 8$.

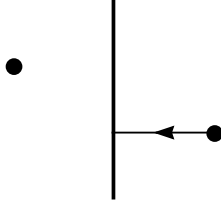[2]This combined interaction is attractive for ND< 4, and repulsive for ND> 4; it vanishes for ND= 4.

Figure 1: A Dp-brane crossing a D$(8-p)$-brane.

The force that is felt by the Dp-brane in the presence of a D$(8-p)$-brane depends on the actual energy of the system, rather than its vacuum energy. In the absence of strings, the force is zero on one side and constant repulsive $-T_0$ on the other. On the other hand, as we have seen above, if we start in a state without any strings and with vanishing force, then a string is created as the branes cross and the force will remain zero.

The process of string creation is equivalent to the effect of Hanany and Witten [16], as can be seen from the following chain of dualities. We start with the configuration of [16]

$$
\begin{aligned}
NS5 &: \ (x^0, x^1, x^2, x^3, x^4, x^5) \\
D5 &: \ (x^0, x^1, x^2, x^7, x^8, x^9) \\
D3 &: \ (x^0, x^1, x^2, x^6) \, .
\end{aligned}
\tag{2.7}
$$

Here each 5-brane induces on the other a charge which is equal to $\frac{1}{2}$ of the charge carried by the end of D3-brane. We T-dualize along (x^1, x^2), thereby mapping the system to

$$
\begin{aligned}
NS5 &: \ (x^0, x^1, x^2, x^3, x^4, x^5) \\
D3 &: \ (x^0, x^7, x^8, x^9) \\
D1 &: \ (x^0, x^6) \, .
\end{aligned}
\tag{2.8}
$$

We then perform a IIB S-duality transformation, giving

$$
\begin{aligned}
D5 &: \ (x^0, x^1, x^2, x^3, x^4, x^5) \\
D3 &: \ (x^0, x^7, x^8, x^9) \\
F1 &: \ (x^0, x^6) \, .
\end{aligned}
\tag{2.9}
$$

This is a particular ND= 8 configuration; all the others are obtained by T-duality. In particular, the D0-D8 system is obtained by T-dualizing along (x^7, x^8, x^9).

The analogy with the Hanany-Witten effect extends also to the induced charges on the D-brane world-volume, which can be derived from the WZW-terms in the world-volume theories. The conservation of these charges requires that a string of appropriate

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orientation is either created or annihilated when the branes cross. On the other hand, it is not yet clear what accounts for the $R(-1)^F$ contribution to the energy. In the following section we shall see that for the special case of the D0-D8 system further constraints give a clearer picture.

3 The D0-D8 System Re-examined

Among the ND= 8 configurations the D0-D8 system is special in two respects:

(i) The D0-brane cannot support a world-volume charge.

(ii) The low-energy effective theory is *massive* IIA supergravity [18, 2], rather than massless IIA or IIB supergravity.

The first point implies that a fundamental string cannot end on an isolated D0-brane [19]. On the other hand, as pointed out in [15], because of (ii) fundamental strings must end on a D0-brane in the presence of D8-branes. Let us briefly review the argument. The action of massive IIA supergravity contains the term

$$\int d^{10}x\sqrt{-g}e^{3\phi/2}[dA^{(1)} + mB^{(2)}]^2 \,, \tag{3.1}$$

which is a generalization of the Higgs mechanism, in which the two-form field $B^{(2)}$ acquires mass m, and $A^{(1)}$ plays the role of the Goldstone boson. In the background of D8-branes $m \neq 0$, and the equation of motion for $B^{(2)}$ gets a contribution from (3.1)

$$d * (e^{-2\phi}dB^{(2)}/2) = m * (dA^{(1)} + mB^{(2)}) \,. \tag{3.2}$$

Integrating the equation over any eight-sphere gives

$$m \int_{s^8} *(dA^{(1)} + mB^{(2)}) = 0 \,. \tag{3.3}$$

If however there is a D-particle carrying R-R charge inside the eight-sphere, this would imply that its flux vanished. To avoid this conclusion, it was argued that fundamental strings must begin or end on the D-particle. This adds a source term $\pm n(2\pi\alpha')^{-1}\delta^{(8)}(x)$ to eq. (3.2), which integrates to give a D0-brane charge[3]

$$\mu_0 = \pm\frac{n}{2\pi\alpha'm} \,. \tag{3.4}$$

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[3] We follow the convention in [15], where the coupling of the string to $B^{(2)}$ is $(2\pi\alpha')^{-1}\int B^{(2)}$.

In [15] m was taken to be equal to $\pm\mu_8$, the unit of D8-brane charge. This reproduces the known relation

$$\mu_0\mu_8 = \frac{1}{2\pi\alpha'} \tag{3.5}$$

for $n = 1$. In general, the number of fundamental strings is therefore $n = |m|/\mu_8$; we shall choose the convention that the string is oriented towards the D-particle if $m > 0$, and away from it if $m < 0$.

However the value of m on either side of the D8-brane is actually $\pm\mu_8/2$, as the jump in m across a single D8-brane is $|\Delta m| = \mu_8$ [20]. For a single D8-brane, the D0-brane is therefore in a region of space with $m = \pm\mu_8/2$, and the appropriate source term would be $(4\pi\alpha')^{-1}\delta^{(8)}(x)$, corresponding to a string with half a fundamental charge (or tension).

The overall orientation of the strings is fixed uniquely by supersymmetry. The D8-brane breaks half the supersymmetry of type IIA string theory, and the D0-brane further breaks half of that, leaving one quarter of the original supersymmetry (eight supersymmetries). A string can be added without breaking any further supersymmetry only if it is perpendicular to the D8-brane and oriented in a particular way (for the $\overline{\text{D8}}$-brane the orientation is opposite). This fixes the configurations uniquely.

For a D0-brane on the left hand side of a D8-brane where $m = -\mu_8/2$, the half string stretches between the D8-brane and the D0-brane and it is oriented in such a way as to go into the D0-brane. On the right hand side we have $m = \mu_8/2$, and the half string comes out of the D0-brane and stretches between the D0- and the D8-brane. (This follows from the fact that the string is oriented the same way on both sides.) As the D0-brane crosses the D8-brane, it takes its half-string with it (which goes into the D0-brane, say). At the same time, a fundamental (whole) string is created which combines with the half-string to give a half-string of the opposite orientation (*i.e.* a half-string which comes out of the D0-brane); we therefore see that the string creation is necessary in order to preserve the supersymmetric orientation of the half-string.
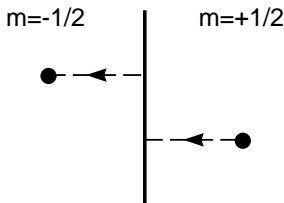

Figure 2: A D0-brane crossing a D8-brane. The dashed line represents a half-string. The value of m is given in units of μ_8.

As half-strings do not seem to be physical, one should really consider the situation with two D8-branes, where all the strings involved are real. The analogous process is

shown in figure 3, where it is assumed that $m = -\mu_8$ on the left, $m = 0$ in the middle, and $m = +\mu_8$ on the right. Thus the force vanishes both on the right and on the left. In the middle the repulsive NS-NS forces due to the two D8-branes cancel, giving again a vanishing force.
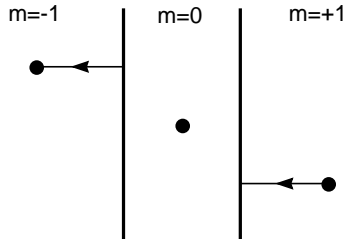

Figure 3: A D0-brane crossing two D8-branes.

This analysis does not extend directly to the other (non-compact) ND= 8 configurations. For example the perpendicular D1-D7 system in type IIB string theory exhibits the same amplitude as our D0-D8 system, but the analogous consistency condition due to the equation of motion for $B^{(2)}$ in massless type IIB supergravity does not require the introduction of fundamental strings. In the compact case, however, the relevant low-energy effective theory is nine-dimensional massive supergravity [21], and the same result is obtained. This is also clear by T-duality.

4 Type IA D-Particles

Type IA string theory is a nine-dimensional string theory which is T-dual to type I theory. It consists of two $\Omega 8$-planes at $x^9 = 0$ and $x^9 = \pi R_{IA}$, and 16 D8-branes and their images. Consider first a D-particle in the presence of a single $\Omega 8$-plane located at $x^9 = 0$. (For the time being, we shall ignore the second orientifold plane and the D8-branes.) If the D-particle is located away from the 8-plane, say at $x^9 = R$, invariance under the combination of world-sheet parity and reflection along x^9 requires the presence of an image D-particle at $x^9 = -R$. The R-dependent contribution to the one-loop vacuum energy of this configuration is given by the one-loop amplitude for the string stretched between the D-particle and its image

$$\mathcal{A} = 2V_1 \int \frac{dk_0}{2\pi} \int_0^\infty \frac{dt}{2t} \mathrm{Tr}\left[e^{-2\pi\alpha' t(k_0^2 + M^2)}(-1)^{F_S}\frac{1}{2}(1+(-1)^F)\frac{1}{2}(1+\Omega I_9) \right], \qquad (4.1)$$

where Ω denotes world-sheet parity, I_9 denotes reflection along x^9,

$$M^2 = \frac{(2R)^2}{4\pi^2\alpha'^2} + \frac{1}{\alpha'}\sum \text{oscillators} + \frac{a}{\alpha'}, \qquad (4.2)$$

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and $a = -1/2$ for the NS-sector, and $a = 0$ in the R-sector. The moding of all the oscillators is as in in (2.3). This amplitude can be thought of as the sum of half an annulus amplitude plus half a Möbius strip amplitude. The factor of 2 outside comes from exchanging the two ends of the string in the annulus case, and from a net of $+2$ even minus odd Chan-Paton factors in the Möbius strip case.

There are eight potential contributions to the amplitude, depending on the fermion moding (R or NS), inclusion or not of $(-1)^F$, and inclusion or not of ΩI_9. The R-sector has fermion zero-modes in all directions, so the $R(-1)^F$ trace vanishes. The action of Ω on the open string modes is given by

$$\Omega : \qquad \begin{matrix} \alpha_n^0 & \longrightarrow & (-1)^n \alpha_n^0 \\ \alpha_n^{1,\dots,9} & \longrightarrow & -(-1)^n \alpha_n^{1,\dots,9} \,, \end{matrix} \qquad (4.3)$$

where there is an extra minus sign for the DD directions compared to the NN direction x^0. The action on the fermionic and the ghosts modes is similar. Together with the obvious action of I_9 this becomes

$$\Omega I_9 : \qquad \begin{matrix} \alpha_n^{0,9} & \longrightarrow & (-1)^n \alpha_n^{0,9} \\ \alpha_n^{1,\dots,8} & \longrightarrow & -(-1)^n \alpha_n^{1,\dots,8} \,. \end{matrix} \qquad (4.4)$$

In particular, the action on the fermionic zero-modes can be represented by

$$(\Omega I_9)_0 = \psi_0^1 \cdots \psi_0^8 \,. \qquad (4.5)$$

As a consequence, the $R(\Omega I_9)$ trace vanishes, but the $R((-1)^F \Omega I_9)$ trace gives a non-vanishing contribution, and the final result for the potential is

$$\begin{aligned} V_{D0-\Omega 8}(R) &= -\int \frac{dt}{4t} (8\pi^2 \alpha' t)^{-1/2} e^{-4R^2 t/(2\pi \alpha')} \times \\ &\qquad \times \left[\frac{f_3^8(q) - f_4^8(q) - f_2^8(q)}{f_1^8(q)} + \frac{-f_4^8(iq) + f_3^8(iq) \pm f_2^8(iq)}{f_2^8(iq)/16} \right] \\ &= 8T_0 R \left[1 \pm 1 \right] \,, \end{aligned} \qquad (4.6)$$

where the first three terms correspond to NS, $NS(-1)^F$ and R, respectively, and the second three terms correspond to $NS(\Omega I_9)$, $NS((-1)^F \Omega I_9)$ and $R((-1)^F \Omega I_9)$, respectively. The first three terms cancel due to the abstruse identity. The sign of the last term corresponds to the choice of the action of $(-1)^F$ on the zero mode part of the R-sector. A modular transformation relates this term to the closed string R-R sector (Appendix B); its sign corresponds thus to the sign of the R-R charge, and therefore to whether we are considering an $\Omega 8$-plane or an $\overline{\Omega 8}$-plane.

The $\Omega 8$-plane is a source of -16 units of D8-brane charge. By symmetry $m = -8\mu_8$ on one side of the 8-plane, and $m = +8\mu_8$ on the other side (for the $\overline{\Omega 8}$-plane the values are reversed). The discussion in section 3 therefore implies that 8 fundamental strings must end on each of the D-particles, with appropriate orientation, as in section 3.
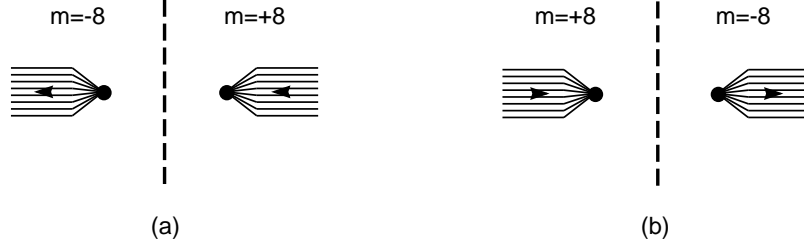
Figure 4: D0-brane and image in the presence of (a) an $\Omega 8$-plane and (b) an $\overline{\Omega 8}$-plane.

5 ND= 8-system with relative velocity

In this section we shall analyze the situation when there is a relative velocity between the two branes, where again ND= 8. For simplicity we shall refer to this system as the D0-D8-brane system, but everything we shall say is also true for all ND= 8 configurations.

There are two different cases to be considered: the D0-brane can move in a direction parallel to the D8-brane, or in a direction transverse to the D8-brane. In the first case, the analysis of section 2 is unchanged, and the potential does not get any velocity dependent corrections. (This is as usual for the case when two D-branes move in a relative world-volume direction.) In the second case, however, there is a velocity dependent correction. Indeed, we shall find that for any non-zero transverse velocity the usual calculation of the phase shift diverges; this divergence is due to the $R(-1)^F$ sector.

The analysis can be done as in [22, 6]. In the present case there is no impact parameter as there is only one transverse direction, and the phase shift is given by

$$\mathcal{A}_{D0-D8}(v) = \int \frac{dt}{4\pi t} \frac{\Theta_1'(0, it)}{\Theta_1(\nu t, it)} f_4^{-8}(q) \left[\frac{\Theta_3(\nu t, it)}{\Theta_3(0, it)} f_2^8(q) - \frac{\Theta_2(\nu t, it)}{\Theta_2(0, it)} f_3^8(q) + J_4 f_4^8(q) \right], \quad (5.1)$$

where $v = \tanh(\nu)$ is the velocity, and J_4 will be explained shortly. The three terms in the bracket correspond to the contributions from the NS, R and $R(-1)^F$ sectors, respectively. The dependence on the distance R can be recovered from the phase shift by identifying $R = v\tau$, where τ is the world-line time.

The contribution of the $R(-1)^F$ sector is unusual. In the static case, the fermions in this sector have a NN and a DD boundary condition in the time and the transverse direction and are hence integer moded. Together with the $(-1)^F$ operator this gives a fermionic zero-mode in the determinants. (Equivalently, this can be seen from the fact that the trace over the ground states gives $(1-1) = 0$.) However, the bosonic superghosts in this sector also have zero-modes, and the two contributions cancel for zero-velocity to give the result in eq. (2.6). Once the D0-brane has a velocity in the transverse direction, the fermions in the time and transverse directions no longer have zero-modes, but the

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superghosts still do; this therefore gives a divergent result

$$J_4 = \infty \times \left(\frac{\Theta_1'(0, it)}{\Theta_1(\nu t, it)}\right)^{-1}. \tag{5.2}$$

In the adiabatic approximation, only the leading velocity correction from the bosonic contribution is considered, and the static potential (with a changing distance) is recovered. Beyond this approximation the potential diverges.

If we ignore the contribution from the $R(-1)^F$ sector, we can compute the velocity dependent potential from the phase shift. We find at long distances

$$V_{long}(R, v) = -\frac{1}{2}T_0 R\left(1 - \frac{1}{2}\frac{v^2}{1 - v^2}\right), \tag{5.3}$$

and in the short distance limit (to leading order in the velocity)

$$V_{short}(R, v) = -\frac{1}{2}T_0 R - \frac{1}{16}T_0\frac{v^2}{R^3}. \tag{5.4}$$

This agrees with the result in [11].

If the D0-brane is between two D8-branes, the two divergences from the two D8-branes cancel. This will always happen when the D0-brane is in a region of spacetime where $m = 0$. For example, the divergence is absent when the cosmological constant induced by the D8-branes is canceled by an $\Omega 8$-plane; in particular, this is the case in the $SO(16) \times SO(16)$ type IA theory.

On the other hand, if $m \neq 0$, the above analysis seems to suggest that the D0-branes cannot move in the transverse direction. For example, let us consider type IIA string theory in the background of a D8-brane (at infinity). This defines a ten-dimensional theory whose low energy effective action is massive type IIA supergravity. If we consider the strong coupling limit of this theory, we might think that an eleventh dimension will open up and that we will get an eleven-dimensional theory with a cosmological constant. The low-energy theory of this theory would be eleven-dimensional supergravity with a cosmological constant, which does not exist [17]. On the other hand, the idea of matrix theory [23] is that in the strong coupling limit of type IIA string theory, everything is made out of D0-branes. Our analysis therefore suggests that none of the states can move in the transverse x^9 direction, and that the resulting theory is again ten-dimensional.

6 World-Line Theories and Matrix Models

We can also study the D0-D8 system from the point of view of the world-line theory of the D-particle. For $R \ll \sqrt{\alpha'}$ this theory is approximately described by the massless

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open string modes, where we only keep the lowest derivatives. The resulting theory is supersymmetric quantum mechanics with 8 supercharges and $Spin(8)$ global R-symmetry. The theory was discussed on general grounds in [24]. The analysis of the short distance degrees of freedom reveals that there is one complex fermion from the $(0-8)$-string, and the relevant quantum mechanics was written down in [11, 12, 24]. However, this description misses a tree-level linear potential which is due to the $R(-1)^F$-term. This potential guarantees that the theory is anomaly free; each complex fermion leads at one-loop to a linear potential and a Chern-Simons term whose coefficients are $\frac{1}{2}$, but the tree-level linear potential (and the tree-level Chern-Simons term required by supersymmetry) has coefficient $\pm\frac{1}{2}$ (for each D8-brane), and this guarantees that the overall coefficient is always integral.

In the notation of [24] the relevant part of the Lagrangian is then (taking $2\pi\alpha' = 1$)

$$\mathcal{L} = \int dt f(\phi)\dot{\phi}^2 - if(\phi)\lambda_a\dot{\lambda}_a \pm \frac{1}{2}(\phi + A_0) - i\bar{\chi}\dot{\chi} - \bar{\chi}(\phi + A_0)\chi\,, \qquad (6.1)$$

where ϕ denotes the x^9 position of the D0-brane, λ_a is its superpartner (an 8-component spinor), A_0 is the world-volume gauge field, and χ is the complex fermion of the D0-D8 string. The supersymmetries and gauge transformation of this Lagrangian were given in [24]. The fact that it is impossible to move the D0-brane along x^9 should be reflected here by the fact that one cannot give ϕ a time dependent expectation value.

We can similarly consider N D0-branes in the presence of a D8-brane. The corresponding world-line theory is described by a supersymmetric $N \times N$ matrix quantum mechanics with 8 supersymmetries. Following the idea of [23], the large N limit of this matrix model should describe the strong coupling behavior of type IIA string theory with a non-trivial D8-brane background. As argued previously, this should represent a ten-dimensional, rather than an eleven-dimensional, theory.

The case of a D0-brane and an Ω8-plane is similar. The effect of the Ω8-plane is to induce an additional linear potential $\pm 8(\phi + A_0)$. The heterotic matrix theory corresponds to the situation where there are additional D8-branes. When eight D8-branes and their images are localized at the Ω8-plane, the linear potentials due to the D8-branes and the orientifold-plane cancel and the world volume theory reduces to that found in [11, 12].

In this case, as discussed in the previous section, the D0-brane can move in the transverse direction (with respect to the D8-branes and the Ω8-plane), as $m = 0$ everywhere. However, when some of the D8-branes are not localized at the orientifold plane, there are regions in space where $m \neq 0$. In such a region there are fundamental strings between the D0-brane and some of the D8-branes, and the D0-branes cannot move non-adiabatically in the transverse direction. The corresponding tree-level potential depends on m, and the world-line theories appropriate for regions of different m are therefore different.

7   Conclusions

In this paper we have discussed configurations of branes with ND= 8. These configurations have peculiar properties. The one-loop open string calculation reveals that there is a R-R interaction between them even though they do not carry the same charge. Furthermore they cannot move non-adiabatically in the direction transverse to both of them, and when they cross each other a fundamental string is created that is stretched between them. The ND= 8 system is U-dual to the system described in [16], and the creation of a fundamental string is U-dual to the creation of a D3-brane when a D5-brane crosses a NS-5-brane.

We have suggested that the inability to move in the transverse direction is related to the fact that there does not exist a eleven-dimensional supergravity theory with a cosmological constant (which would be induced by nine-branes).

We have shown that a system of two branes with ND= 8 either has a vanishing force, or a force which is twice the NS-NS force. The same also holds for the system of a Dp-brane and an $\Omega(8 - p)$-plane. If we start with a system where the force vanishes, then the string creation process guarantees that the force remains zero after the branes have crossed. For the special case of an isolated D0-D8 system, further constraints imply that the force always vanishes. Our results are therefore consistent with the previous analysis of these systems [11, 12, 13] for the case where eight D8-branes and their images are at each Ω8-plane; the results differ, however, for other configurations.

There are many things that need to be understood further.

(i) The precise finite velocity dynamics and its implications need to be understood better.

(ii) For systems with ND= 8 other than the D0-D8-system, the nature of the interaction which is due to the R-R sector is rather unclear. In particular, the corresponding supergravity theories do not seem to require that fundamental strings have to be present for consistency. On the other hand, the corresponding branes *can* support world-volume charges.

Let us take the D1-D7 system as an example. On each of the branes there is a coupling of the R-R charge of the other brane to the world-volume gauge potential, and also to a pull back of the $B^{NS}_{\mu\nu}$ field to the world-volume of the brane. There is also a coupling in the supergravity of the form $\chi dB^{NS} dB^R$, where χ is the axion and B is a two-form potential that couples to the D-string or the elementary string. A combination of these may be responsible for the extra interaction.

(iii) The results of this paper may have implications for the description of the duality between type IA and the heterotic string theory. For instance the creation of a

string as a D8-brane crosses a D0-brane which is stuck at the $\Omega 8$-plane may be related to the momentum modes which appear on the heterotic side at a point of enhanced gauge symmetry [20, 5].

(iv) More speculative is the connection to the heterotic matrix theory. It is known that upon compactification on S^1, turning on Wilson lines seems to create an anomaly. It was suggested in [25] that this may be canceled with an anomaly inflow from the bulk. It was shown in [30] that the anomaly inflow has to do with the creation of fundamental strings, which, as we have seen, is connected to an extra term in the interaction of D0-branes and D8-branes (or an $\Omega 8$-plane). Taking this extra interaction into account may help to resolve the problem of compactifications.

These issues are currently under study.

Acknowledgments

Appendix: Closed String Calculations

A   D0-D8

In this section we shall explain how the results can also be obtained by performing a closed string calculation. We shall work covariantly, and we shall use the same conventions as in [26] (see also [27]).

We shall first describe boundary states which satisfy Neumann boundary conditions for x^μ with $\mu = 0, \ldots, p$, and Dirichlet boundary conditions for x^i with $i = p + 1, \ldots, 9$. We shall then also consider the boundary states which can be obtained from these by Lorentz transformations. We shall work in $d = 10$ spacetime dimensions, and we denote by $d^\perp$ the number of transverse dimensions, *i.e.* $d^\perp = 9 - p$.

Let us first consider boundary states which are localized at the transverse position y^i. These can be described as linear combinations of coherent states of the form

$$|Bp, y^i, \eta\rangle = |Bp, y^i\rangle_b \, |Bp, \eta\rangle_f \, |B, \eta\rangle_g \, , \tag{A.1}$$

where the subscripts b, f and g denote the bosonic, fermionic and ghost sector, respectively. The component in the bosonic sector $|Bp, y^i\rangle_b$ is

$$(2\pi\sqrt{\alpha'})^{d^\perp} \prod_{i=p+1}^{9} \delta(q^i - y^i) \exp\left\{\sum_{n=1}^{\infty} \frac{1}{n}\left[-\eta_{\mu\nu}\alpha^\mu_{-n}\tilde{\alpha}^\nu_{-n} + \alpha^i_{-n}\tilde{\alpha}^i_{-n}\right]\right\}|0\rangle, \qquad (A.2)$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$, and $|0\rangle$ is the zero-momentum ground state. Here the sum over μ and ν runs over $0, \ldots, p$, and the sum over i is from $p+1$ to 9. The other two components depend on whether we are considering the NS-NS sector or the R-R sector. In the first case, the fermionic state $|Bp, \eta\rangle_f$ is

$$|Bp, \eta\rangle_{NSNS} = \exp\left\{i\eta \sum_{r>0}\left[-\eta_{\mu\nu}\psi^\mu_{-r}\tilde{\psi}^\nu_{-r} + \psi^i_{-r}\tilde{\psi}^i_{-r}\right]\right\}|0\rangle_{NSNS}, \qquad (A.3)$$

where $|0\rangle_{NSNS}$ is the NS-NS ground state, and in the second case

$$|Bp, \eta\rangle_{RR} = \exp\left\{i\eta \sum_{m=1}^{\infty}\left[-\eta_{\mu\nu}\psi^\nu_{-m}\tilde{\psi}^\nu_{-m} + \psi^i_{-m}\tilde{\psi}^i_{-m}\right]\right\}|p, \eta\rangle^0_{RR}, \qquad (A.4)$$

where $|p, \eta\rangle^0_{RR}$ is the R-R ground state satisfying

$$\psi^\mu_{\mp}|p, \pm\rangle^0_{RR} = \psi^i_{\pm}|p, \pm\rangle^0_{RR} = 0, \qquad \psi^\alpha_\pm \equiv \frac{1}{\sqrt{2}}\left(\psi^\alpha_0 \pm i\tilde{\psi}^\alpha_0\right). \qquad (A.5)$$

The coherent state in the ghost sector $|B\eta\rangle_g$ does not depend on p, and is as given in [28]. As explained in [9] (see also [28, 26]), invariance under the GSO projection (and the consistency with the open string sector) requires that the physical D-brane states are linear combinations of these states

$$|Dp, y^i\rangle = |Dp, y^i\rangle_{NSNS} + |Dp, y^i\rangle_{RR}, \qquad (A.6)$$

where

$$|Dp, y^i\rangle_{NSNS} = \frac{\mathcal{N}_{NSNS}}{2}\left(|Bp, y^i, +\rangle_{NSNS} - |Bp, y^i, -\rangle_{NSNS}\right) \qquad (A.7)$$

is the component in the NS-NS sector, and

$$|Dp, y^i\rangle_{RR} = \frac{\mathcal{N}_{RR}}{2}\left(|Bp, y^i, +\rangle_{RR} + |Bp, y^i, -\rangle_{RR}\right) \qquad (A.8)$$

is the component in the R-R sector. The GSO condition also implies that p is even for type IIA, odd for type IIB, and $p = 1, 5, 9$ in type I. We normalize the ground states so that $_{NSNS}\langle 0|0\rangle_{NSNS} = 1$, and

$$_{RR}^{0}\langle p', \eta'|p, \eta\rangle^0_{RR} = \delta_{p,p'}\delta_{\eta,\eta'} + \delta_{|p-p'|,10}\delta_{\eta,-\eta'}. \qquad (A.9)$$

The normalization constants $\mathcal{N}_{NSNS}$ and $\mathcal{N}_{RR}$ are then determined up to a sign by consistency with the open string calculation. As the overall sign is irrelevant, there exist two different solutions which differ by the relative sign, and they correspond to the brane and the anti-brane solution, respectively.

The amplitude to propagate from one D-brane state to another is given, up to a normalization, by

$$\mathcal{A}_{Dp-Dp'} = \int_0^\infty \frac{d\tau}{\tau} \langle Dp, y | e^{-\pi\tau(L_0 + \tilde{L}_0)} | Dp', y' \rangle . \tag{A.10}$$

To fix the normalization constants we shall first determine the amplitude between two stationary p-branes. Using the conventions of [26] for the normalization of the amplitude (including the same normalization factor of $(2\alpha')^{-d/2}/\pi$) we find for the contribution in the NS-NS sector

$$\mathcal{A}_{Dp-Dp}^{NSNS} = \frac{\mathcal{N}_{NSNS}^2}{2} \frac{V_{p+1}}{(8\pi^2\alpha')^{(p+1)/2}} \int_0^\infty d\tau \tau^{-4+(p-1)/2} e^{-R^2/2\pi\alpha'\tau} \frac{f_3^8(r) - f_4^8(r)}{f_1^8(r)}, \tag{A.11}$$

where R is the transverse distance between the two p-branes, and $r = e^{-\pi\tau}$; in the R-R sector we get

$$\mathcal{A}_{Dp-Dp}^{RR} = \frac{\mathcal{N}_{RR}^2}{32} \frac{V_{p+1}}{(8\pi^2\alpha')^{(p+1)/2}} \int_0^\infty d\tau \tau^{-4+(p-1)/2} e^{-R^2/2\pi\alpha'\tau} \frac{f_2^8(r)}{f_1^8(r)} . \tag{A.12}$$

Here the functions f_i are defined as

$$
\begin{aligned}
f_1(q) &= q^{1/12} \prod_{n=1}^\infty (1 - q^{2n}) \\
f_2(q) &= \sqrt{2} q^{1/12} \prod_{n=1}^\infty (1 + q^{2n}) \\
f_3(q) &= q^{-1/24} \prod_{n=1}^\infty (1 + q^{2n-1}) \\
f_4(q) &= q^{-1/24} \prod_{n=1}^\infty (1 - q^{2n-1}) .
\end{aligned}
\tag{A.13}
$$

It follows that the total amplitude $\mathcal{A}_{Dp-Dp}^{NSNS} + \mathcal{A}_{Dp-Dp}^{RR}$ vanishes provided that $\mathcal{N}_{RR} = 4i\mathcal{N}_{NSNS}$. To fix the remaining overall constant, we substitute $t = 1/\tau$, and perform a modular transformation to obtain

$$\mathcal{A}_{Dp-Dp}^{NSNS} = \frac{\mathcal{N}_{NSNS}^2}{2} \frac{V_{p+1}}{(8\pi^2\alpha')^{(p+1)/2}} \int_0^\infty \frac{dt}{t} t^{-(p+1)/2} e^{-R^2 t/2\pi\alpha'} \frac{f_3^8(q) - f_2^8(q)}{f_1^8(q)}, \tag{A.14}$$

where $q = e^{-\pi t}$. This agrees with the open string calculation provided we set $\mathcal{N}_{NSNS} = 1$.

Now that all normalization constants are determined, we can calculate the amplitude for a stationary D0-D8 configuration. In the NS-NS sector we find by a similar calculation

$$\mathcal{A}_{D0-D8}^{NSNS} = \frac{V_1}{(8\pi^2\alpha')^{1/2}} \frac{1}{2} \int_0^\infty d\tau \tau^{-1/2} e^{-R^2/2\pi\alpha'\tau} \frac{f_4^8(r) - f_3^8(r)}{f_2^8(r)}, \qquad (A.15)$$

where again $r = e^{-\pi\tau}$. Using the abstruse identity it follows that the oscillator contribution (the ratio of the f-functions) equals -1. Substituting $t = 1/\tau$ and performing a modular transformation we then find

$$
\begin{aligned}
\mathcal{A}_{D0-D8}^{NSNS} &= \frac{1}{2} \frac{V_1}{(8\pi^2\alpha')^{1/2}} \int_0^\infty \frac{dt}{t} t^{-1/2} e^{-R^2 t/2\pi\alpha'} (-1) \\
&= -\frac{1}{2} \frac{V_1}{(2\pi\alpha')} R \frac{\Gamma(-1/2)}{2\sqrt{\pi}} = \frac{1}{2} \frac{V_1}{(2\pi\alpha')} R, \qquad (A.16)
\end{aligned}
$$

which agrees with the open string calculation, and reproduces the crucial normalization factor. Superficially, the contribution from the R-R sector vanishes since the R-R ground states only contribute for $p = p'$ and $|p - p'| = 10$ as follows from (A.9). However, the calculation is more subtle as the contribution from the superghost ground state has to be taken into account. Indeed, the contribution of the superghosts to the amplitude $\langle B0, y_0, \eta | e^{-\pi\tau(L_0 + \tilde{L}_0)} | B8, y_8, -\eta \rangle$ is

$$\prod_n (1 - r^{2n})^{-2}, \qquad (A.17)$$

and thus their ground state gives a divergent term $(n = 0)$, which can potentially cancel the vanishing R-R ground state amplitude. In fact, it is clear from the gauge fixing condition for the light cone gauge, that there are only eight physical zero modes, and the light-cone calculation (see for example [9]) then implies that there is a non-vanishing contribution from the R-R sector for $|p - p'| = 8$. This contribution cancels precisely the NS-NS contribution if one boundary state is a brane, and one an anti-brane, and doubles the NS-NS contribution in the case where both boundary states are branes or anti-branes.

Next we shall consider the boundary states that can be obtained from the stationary boundary states by the application of a Lorentz transformation. We want to analyze first the case, where the Lorentz transformation corresponds to a pure space-rotation; in this case, we want to show that a rotation by π in a plane which is spanned by one Neumann and one Dirichlet direction turns a p-brane into an anti-p-brane, whereas a rotation by π in a plane spanned by two Neumann or two Dirichlet directions leaves the p-brane invariant.

Let us recall [29] that the generator of the Lorentz transformation is given as

$$J^{\mu\nu} = l^{\mu\nu} + E^{\mu\nu} + K^{\mu\nu}, \tag{A.18}$$

where $l^{\mu\nu} = q^\mu p^\nu - q^\nu p^\mu$,

$$E^{\mu\nu} = -i \sum_{n=1}^{\infty} \frac{1}{n} \left(\alpha_{-n}^\mu \alpha_n^\nu - \alpha_{-n}^\nu \alpha_n^\mu + \tilde{\alpha}_{-n}^\mu \tilde{\alpha}_n^\nu - \tilde{\alpha}_{-n}^\nu \tilde{\alpha}_n^\mu \right), \tag{A.19}$$

and

$$K^{\mu\nu} = -i \sum_{r>0} \left(\psi_{-r}^\mu \psi_r^\nu - \psi_{-r}^\nu \psi_r^\mu + \tilde{\psi}_{-r}^\mu \tilde{\psi}_r^\nu - \tilde{\psi}_{-r}^\nu \tilde{\psi}_r^\mu \right) \tag{A.20}$$

in the NS-NS sector, and

$$K^{\mu\nu} = -\frac{i}{2} \left([\psi_0^\mu, \psi_0^\nu] + [\tilde{\psi}_0^\mu, \tilde{\psi}_0^\nu] \right) - i \sum_{m=1}^{\infty} \left(\psi_{-m}^\mu \psi_m^\nu - \psi_{-m}^\nu \psi_m^\mu + \tilde{\psi}_{-m}^\mu \tilde{\psi}_m^\nu - \tilde{\psi}_{-m}^\nu \tilde{\psi}_m^\mu \right) \tag{A.21}$$

in the R-R sector. A finite Lorentz transformation is then of the form

$$U(\alpha) = \exp\left(i\theta_{\mu\nu} J^{\mu\nu} \right). \tag{A.22}$$

We shall consider a spatial rotation in the $i-j$ plane, and this corresponds to the situation where $\theta_{\mu\nu} = 0$ unless $(\mu, \nu) = (i, j)$. It is then easy to check that

$$U(\theta_{ij})\alpha_{-n}^k U(\theta_{ij})^{-1} = \begin{cases} \alpha_{-n}^k & \text{if } k \neq i, j, \\ \cos(\theta_{ij})\alpha_{-n}^i - \sin(\theta_{ij})\alpha_{-n}^j & \text{if } k = i, \\ \cos(\theta_{ij})\alpha_{-n}^j + \sin(\theta_{ij})\alpha_{-n}^i & \text{if } k = j. \end{cases} \tag{A.23}$$

If we take $\theta_{ij} = \pi$, it then follows that, apart from the bosonic zero modes which transform in the obvious way, the bosonic sector and the NS-NS sector of the boundary state are invariant under $U(\pi)$. Similarly the oscillator part of the R-R sector is also invariant. The action on the R-R ground state however depends on the boundary conditions in the plane of rotation (i, j). The relevant part of $U(\pi)$ is $\exp\left(\pi(\psi_+^i \psi_-^j + \psi_-^i \psi_+^j) \right)$, whose action gives

$$\exp\left(\pi(\psi_+^i \psi_-^j + \psi_-^i \psi_+^j) \right) |p, \eta\rangle_{RR}^0 = \begin{cases} |p, \eta\rangle_{RR}^0 & \text{for (N,N) or (D,D)} \\ -|p, \eta\rangle_{RR}^0 & \text{for (N,D) or (D,N).} \end{cases} \tag{A.24}$$

Such a rotation therefore changes the sign of the R-R component, and thus transforms a p-brane into an anti-p-brane and vice versa.

Next we shall analyze the effect of a boost on the boundary state. Following the analysis of [26], we find that a boost in the k-th direction with rapidity ν (where $k \geq p+1$) transforms the boundary state $|p, y^i, \eta\rangle$ to

$$|Bp, y^i, v, \eta\rangle = |Bp, y^i, v\rangle_b |Bp, \eta, v\rangle_f |B, \eta\rangle_g, \tag{A.25}$$

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where $|Bp, y^i, v\rangle_b$ is now

$$
\begin{aligned}
|Bp, y^i, v\rangle_b \;=\;& (\alpha')^{d^\perp/2} \sqrt{1 - v^2}\, \delta(q^k - q^0 v - y^k) \prod_{i \neq k} \delta(q^i - y^i) \\
& \exp\left\{ \sum_{n=1}^\infty \frac{1}{n} \left[-\sum_{\nu=1}^p \alpha_{-n}^\nu \tilde{\alpha}_{-n}^\nu + \sum_{i \neq k} \alpha_{-n}^i \tilde{\alpha}_{-n}^i \right] \right\} \\
& \exp\left\{ \sum_{n=1}^\infty \frac{1}{n} + \cosh(2\nu) \left(\alpha_{-n}^0 \tilde{\alpha}_{-n}^0 + \alpha_{-n}^k \tilde{\alpha}_{-n}^k \right) \right\} \\
& \exp\left\{ \sum_{n=1}^\infty \frac{1}{n} \left[\sinh(2\nu) \left(\alpha_{-n}^0 \tilde{\alpha}_{-n}^k + \alpha_{-n}^k \tilde{\alpha}_{-n}^0 \right) \right] \right\} |0\rangle , \qquad (A.26)
\end{aligned}
$$

and $v = \tanh(\nu)$ is the velocity. A similar formula holds for the NS-NS fermions, and for the non-zero oscillators in the R-R sector. The R-R ground state on the other hand becomes

$$
|p, \pm, v\rangle_{RR}^0 = \cosh(\nu)|p, \pm\rangle_{RR}^0 + \sinh(\nu)\psi_\pm^0 \psi_\mp^k |p, \pm\rangle_{RR}^0 . \qquad (A.27)
$$

Using these expressions we can evaluate the amplitude for the configuration of a static 8-brane, and a 0-brane with velocity $\tanh(\nu)$ in the x^9 direction. Using the same normalization as before we obtain in the NS-NS sector

$$
\begin{aligned}
\mathcal{A}_{D0-D8}^{NSNS}(v) \;=\;& \frac{1}{4} \frac{1}{\sinh(\nu)} \int_0^\infty d\tau \frac{1}{f_2^8(r)} \prod_{n=1}^\infty \frac{(1 - r^{2n})^2}{(1 - e^{2\nu} r^{2n})(1 - e^{-2\nu} r^{2n})} \\
& \left(f_4^8(r) \prod_{n=1}^\infty \frac{(1 + e^{2\nu} r^{2n-1})(1 + e^{-2\nu} r^{2n-1})}{(1 + r^{2n})^2} \right. \\
& \left. - f_3^8(r) \prod_{n=1}^\infty \frac{(1 - e^{2\nu} r^{2n-1})(1 - e^{-2\nu} r^{2n-1})}{(1 - r^{2n})^2} \right) , \qquad (A.28)
\end{aligned}
$$

where $r = e^{-\pi\tau}$. In the R-R sector, it follows from (A.27), together with (A.9) and

$$
{}_{RR}^{\;\;0}\langle 0, \eta_0 | \psi_\pm^0 \psi_\mp^9 | 8, \eta_8 \rangle_{RR}^0 = -\delta_{\eta_0, -\eta_8} , \qquad (A.29)
$$

that the R-R component of the amplitude vanishes unless $\eta_8 = -\eta_0$, and the contribution is then proportional to $\sinh(\nu)$. In addition we find that all oscillator contributions cancel, and the amplitude becomes

$$
\mathcal{A}_{D0-D8}^{RR}(v) = -\frac{SG_0}{4} \int_0^\infty d\tau , \qquad (A.30)
$$

where we have represented by SG_0 the divergent contribution of the superghost ground state of $\langle \eta | e^{-\pi\tau(L_0 + \tilde{L}_0)} | -\eta \rangle_g$. The R-R amplitude is independent of the velocity, and thus persists in the limit $\nu \to 0$. This is necessary in order to cancel the $\sinh(\nu)^{-1}$ divergence of the NS-NS amplitude in this limit.

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B D0-Ω8

Just as the original boundary states of Polchinski and Cai [28] can be generalized to boundary states describing Dp-branes, one can generalize the crosscap states. Indeed, it is natural to introduce the concept of a p-crosscap as the state which satisfies the conditions

$$
\begin{aligned}
(\alpha_n^\mu + e^{i\pi n}\widetilde{\alpha}_{-n}^\mu)\,|Cp,\eta\rangle = (\psi_m^\mu + \eta e^{i\pi m}\widetilde{\psi}_{-m}^\mu)\,|Cp,\eta\rangle &= 0 && \mu = 0,\dots,p, \\
(\alpha_n^i - e^{i\pi n}\widetilde{\alpha}_{-n}^i)\,|Cp,\eta\rangle = (\psi_m^i - \eta e^{i\pi m}\widetilde{\psi}_{-m}^i)\,|Cp,\eta\rangle &= 0 && i = p+1,\dots,9 \,.
\end{aligned} \tag{B.1}
$$

Up to normalization, the unique solution is

$$
|Cp,y^i,\eta\rangle = |Cp,y^i\rangle_b\,|Cp,\eta\rangle_f\,|C,\eta\rangle_g\,, \tag{B.2}
$$

where $|Cp,y^i\rangle_b$ is

$$
(2\pi\sqrt{\alpha'})^{d^\perp}\prod_{i=p+1}^{9}\delta(q^i - y^i)\exp\left\{\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\left[-\eta_{\mu\nu}\alpha_{-n}^\mu\tilde{\alpha}_{-n}^\nu + \alpha_{-n}^i\tilde{\alpha}_{-n}^i\right]\right\}|0\rangle\,. \tag{B.3}
$$

The fermionic contribution in the NS-NS sector is

$$
|Cp,\eta\rangle_{NSNS} = \exp\left\{i\eta\sum_{r>0}e^{i\pi r}\left[-\eta_{\mu\nu}\psi_{-r}^\mu\tilde{\psi}_{-r}^\nu + \psi_{-r}^i\tilde{\psi}_{-r}^i\right]\right\}|0\rangle_{NSNS}\,, \tag{B.4}
$$

and the contribution in the R-R sector is

$$
|Cp,\eta\rangle_{RR} = \exp\left\{i\eta\sum_{m=1}^{\infty}(-1)^m\left[-\eta_{\mu\nu}\psi_{-m}^\mu\tilde{\psi}_{-m}^\nu + \psi_{-m}^i\tilde{\psi}_{-m}^i\right]\right\}|p,\eta\rangle_{RR}^0\,, \tag{B.5}
$$

where $|p,\eta\rangle_R$ is the same ground state as defined before, since it satisfies the same equations as before, i.e. (B.1) with $m = 0$. The ghost sector state $|C\eta\rangle_g$ is independent of p, and is given as in [28].

As before, invariance under the GSO projection and consistency with the open string sector restrict the physical orientifold plane (Ω-plane) states to be

$$
|\Omega p\rangle = \frac{\mathcal{N}_{NSNS}^C}{2}\Big(|Cp,+\rangle_{NSNS} - |Cp,-\rangle_{NSNS}\Big) \pm \frac{\mathcal{N}_{RR}^C}{2}\Big(|Cp,+\rangle_{RR} + |Cp,-\rangle_{RR}\Big), \tag{B.6}
$$

where the normalization constants are fixed by consistency with the open string Möbius strip to be $\mathcal{N}_{NSNS}^C = 2^{p-4}i$ and $\mathcal{N}_{RR}^C = 2^{p-2}i$. The choice of sign reflects the fact that there exist anti-orientifold planes as well as orientifold planes; this is in complete analogy to the situation for Dirichlet branes.

The stationary amplitude between a Dp-brane and an $\Omega p'$-plane can be calculated in the same way as the amplitude of two D-branes. The NS-NS contribution is given by (assuming $p' > p$)

$$\mathcal{A}_{Dp-\Omega p'}^{NSNS} = C \int_0^\infty d\tau \tau^{(p'-9)/2} e^{-R^2/2\pi\alpha'\tau} \frac{f_3(ir)^{8+p-p'} f_4(ir)^{p'-p} - f_4(ir)^{8+p-p'} f_3(ir)^{p'-p}}{f_1(ir)^{8+p-p'} f_2(ir)^{p'-p}},$$
(B.7)

and the R-R contribution is

$$\mathcal{A}_{Dp-\Omega p'}^{RR} = C \int_0^\infty d\tau \tau^{(p'-9)/2} e^{-R^2/2\pi\alpha'\tau} \left[\frac{f_2^8(ir)}{f_1^8(ir)} \delta_{p,p'} + \delta_{p,p'-8} \right].$$
(B.8)

The overall constant C is the same in both cases, and is given by

$$C = -2^{p'-5} (8\pi^2\alpha')^{(p'-9)/2} V_1 .$$
(B.9)

For D0-Ω8 this gives

$$
\begin{aligned}
\mathcal{A}_{D0-\Omega 8}^{NSNS} &= 8 \frac{V_1}{(8\pi^2\alpha')^{1/2}} \int_0^\infty d\tau \tau^{-1/2} e^{-R^2/2\pi\alpha'\tau} \\
&= -8 \frac{V_1}{(2\pi\alpha')} R ,
\end{aligned}
$$
(B.10)

and the same for the R-R contribution. As in the D0-D8 case, there are two choices for the relative sign between the NS-NS and R-R contributions, one corresponding to an Ω8-plane and the other to an $\overline{\Omega 8}$-plane. In one case the total amplitude vanishes, and in the other it is twice (B.10).

Similarly, the amplitude describing a static Ω8-plane and a D0-brane moving transverse to it is found to be

$$
\begin{aligned}
\mathcal{A}_{D0-\Omega 8}^{NSNS}(v) &= 4 \frac{1}{\sinh(v)} \int_0^\infty d\tau \frac{1}{f_2^8(ir)} \prod_{n=1}^\infty \frac{(1-(ir)^{2n})^2}{(1-e^{2v}(ir)^{2n})(1-e^{-2v}(ir)^{2n})} \\
&\quad \left(f_4^8(ir) \prod_{n=1}^\infty \frac{(1+e^{2v}(ir)^{2n-1})(1+e^{-2v}(ir)^{2n-1})}{(1+(ir)^{2n})^2} \right. \\
&\quad \left. - f_3^8(ir) \prod_{n=1}^\infty \frac{(1-e^{2v}(ir)^{2n-1})(1-e^{-2v}(ir)^{2n-1})}{(1-(ir)^{2n})^2} \right) ,
\end{aligned}
$$
(B.11)

and the corresponding R-R amplitude is

$$\mathcal{A}_{D0-\Omega 8}^{RR}(v) = -4(SG_0) \int_0^\infty d\tau ,$$
(B.12)

where SG_0 is again the divergent contribution from the superghost ground states.

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